

Design an Instruction with Mathematical Modeling Based on Scientific-Humanistic Perspectives

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Abstract

Mathematical activities that utilize mathematical viewing and thinking are in demand. In this paper, we have organized the correspondence between mathematical activities and realistic theories of mathematics education. Furthermore, based on the suggestion in previous research that the mathematical way of thinking has not been clearly defined and that by clarifying it, we can find out what children are like in its definition, we have clarified and classified the mathematical ways of thinking that should be emphasized in mathematics education theory. As a result, it was found that they can be classified into two categories: abstract and integrative/developmental.

Keywords: Mathematical viewing and thinking, mathematics education, RME theory, mathematical activities

Introduction

In the Courses of Study for Senior High School (The Ministry of Education, Culture, Sports, Science, and Technology, 2018), the mathematics course aims to cultivate the ability to think mathematically through mathematical activities using mathematical thinking. The revised version of the textbook states that mathematical thinking is to grasp the characteristics and essence of events by focusing on concepts such as quantities, figures, and the relationships between them. In this revision, emphasis is placed on thinking about events in an integrated and developed manner.

Integrative thinking is a way of thinking that abstracts and uses the essential commonalities among many things from a broader perspective (Katagiri, 1988). Itakura (1973) classifies integrative thinking into two categories. One is creating a new concept or theory by focusing on the commonality of the structure of independent concepts and integrating them. The other is to think of new matters from existing principles and create something that encompasses them. Developmental thinking is using what has been integrated with a broader scope, seeking a better method even after a result has been obtained, and discovering something new and more general, based on this (Katagiri, 1988).

According to the mathematics courses of study (MEXT, 2018), mathematical activity is to perceive events mathematically, find mathematical problems, and solve them independently and collaboratively. The process of mathematical activity can be captured in two processes: finding and solving problems in the real world and in the mathematical world. Koderu (2018) states that the process of problem discovery and solution in the real world shows the possibility of creating new mathematics, based on the concepts found in the problem discovery and solution in the real world. In the above Courses of Study, mathematical viewing and thinking are considered essential for creative mathematics learning. However, Kurosawa (2020) points out that the meaning of mathematical thinking is not clearly defined.

Contrarily, realistic mathematics education (RME) theory is a theory of mathematics education related to real-world problems (Gravemeijer and Doorman, 1999). Ikeda (2017) explains that RME theory is based on the concept

of "horizontal mathematization" and "vertical mathematization" as the core activities for problems that students can empirically imagine. The RME theory (Ikeda, 2017) is based on the two core activities of "horizontal mathematization" and "vertical mathematization. Horizontal mathematization is the process of transforming a problem outside of arithmetic and mathematics into a mathematically tractable problem. Vertical mathematization refers to activities that treat and generalize arithmetic and mathematics as tools. The transition from horizontal to vertical mathematization is the process of constructing mathematical knowledge. Sato (2018) states that the idea of RME theory is similar to the idea of mathematics education in Japan.

Based on the above, it is necessary to show the relationship between mathematical activities and RME theory, consider the mathematical viewing and thinking at work, and provide concrete descriptions of these activities.

Purpose

This study aims to show the relationship between mathematical activities and RME theory and classify the mathematical viewing and thinking at work therein.

To this end, we analyze the content of mathematical activities and activities in RME theory, and show their correspondence. Next, based on the mathematical thinking stipulated in previous studies, the mathematical viewing and thinking at work in these activities will be clarified.

Method

Based on previous research, we will map mathematical activities to RME theory. First, "horizontal mathematization" in RME theory is considered to correspond to the process of finding and solving problems in the real world in mathematical activities. Conversely, "vertical mathematization" is viewed as the process of finding and solving problems in the mathematical world in mathematical activities.

We will discuss what kind of mathematical viewing and thinking should be emphasized for these two "mathematizations" and the problem discovery/solving process. In doing so, we will categorize them based on previous research on mathematical viewing and thinking.

Results and Discussion

First, in "horizontal mathematization," the thinking of abstraction is thought to be at work in transforming problems outside of arithmetic and mathematics into mathematically tractable problems. Abstraction is the act of clarifying and concisely expressing an action that was intuitive and ambiguous according to its purpose or gutting a skeletal structure out of multiple events (Ikeda, 2017). According to Katagiri (1988), abstraction can abstract properties from everyday problem situations to create mathematically tractable problems. Furthermore, abstraction requires the concepts of "concretization," which involves segregating events based on abstracted properties, and "idealization," which considers an ideal state in which conditions and properties satisfy mathematical definitions and laws of principle, and clarification of conditions.

Next, many mathematical thoughts are at work in "vertical mathematization." However, if we consider the transition from "horizontal mathematization" to "vertical mathematization," it is the integrative and developmental thinking that we should focus on. **Table 1** shows the correspondence between mathematical activities, mathematical viewing and thinking, and RME theory.

Table 1

Mathematical viewing and thinking that should be emphasized

	Horizontal Mathematization	Vertical Mathematization
Mathematical Activities	Problem finding and solving process in the real world	Problem finding and solving process in the world of mathematics
Mathematical Viewing and Thinking	Concept of Abstraction • Concretization • Idealization • Clarification of conditions	Integrative Thinking Developmental Thinking

Conclusion

In this paper, we believe it is necessary to clarify the role of mathematical viewing and thinking in RME theory based on previous research. We have organized and discussed the mathematical viewing and thinking in RME theory.

In the future, it will be necessary to design, implement, and evaluate lessons on RME theory based on the contents of this paper while keeping in mind the need to utilize mathematical viewing and thinking.

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