Design and Evaluation of Lessons Incorporating Mathematical Activities for Special Cases to Improve Spatial Visualization Ability

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Abstract

High School Mathematics in Japan is required to improve spatial ability through mathematical activities. The improvement of spatial ability is especially related to the improvement of spatial visualization ability, which can be greatly enhanced by augmented reality (AR) technology. AR is a technology that can augment the real world as seen by humans through adding, deleting, emphasizing, or attenuating information in the real environment. In contrast, it is desirable to examine the effects of technology utilization separately for technological and conceptual mathematical activities. Therefore, this study aimed to design and evaluate lessons incorporating mathematical activities for special cases to improve spatial visualization ability. This paper will discuss AR application and learning materials and tests, which are elements of the lessons.

Keywords: AR, Instructional Design, Mathematics Education, Secondary Education, Spatial Ability

Introduction

According to the Courses of Study in Japan, "Properties of Shapes" in high school mathematics requires students to improve spatial ability through mathematical activities. The mathematical activities are to understand events mathematically, find mathematical problems, and solve the problems independently and collaboratively (Ministry of Education, Culture, Sports, Science, and Technology 2018).

Garderen (2006) believes that the improvement of spatial visualization ability is particularly relevant to the improvement of spatial ability. Furthermore, Omar et al. (2019) argued that mobile-augmented reality (mAR) technology improved spatial visualization ability significantly. The mAR is considered part of AR where virtual objects are displayed on a mobile device instead of a PC or head-mounted display (Jamili et al. 2013).

When examining the effectiveness of mAR in mathematics education, researchers should introduce mathematical activities using technology (Estapa & Nadolny 2015). Mathematical activities using the technology have technical and conceptual mathematical activities (Zbiek et al. 2007). These activities are recommended to verify the effectiveness and relevance of technologies such as mAR, based on a clear distinction between them.

According to Zbiek et al. (2007), there are two types of technical and conceptual mathematical activities. The technical mathematical activities are to take mathematical actions on mathematical objects or representations of those objects. A procedure can then be created from a sequence of mathematical actions (or from a previously created procedure). Examples of technical mathematical activities include geometric construction and measurement, numerical computation, algebraic manipulation, graphing, graphical transformation, conversion between notations, solving equations, creating, displaying, collecting, and sorting diagrams. The conceptual mathematical activities involve understanding, communicating, and using mathematical relationships, structures, and relations. Examples of conceptual mathematical activities include finding and describing patterns (inductive reasoning), defining, inferring, generalizing, abstracting, connecting representations, predicting, testing, proving, and refuting. The teachers should create a technical and conceptual distinction between lesson designs to cultivate each of these elements.

Based on the above, this study aimed to design and evaluate lessons incorporating technical and conceptual mathematical activities to improve spatial visualization ability. This paper will discuss the mAR and learning materials and tests, which are elements of the lessons.

mAR Overview

Features and Functions of mAR

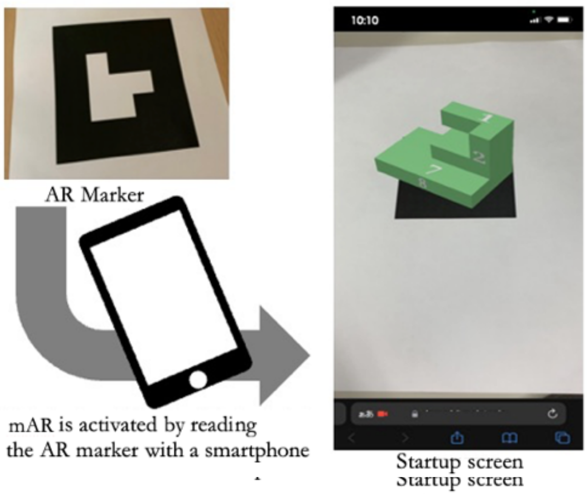
In this study, we used the mAR Hosono et al. (2021) developed. The mAR has a marker tracking function that displays an object simply by reading a specified marker. As an additional function, the mAR can read data in glTF format from three-dimensional (3D) objects created by an object construction application (Tinkercad) and automatically issue AR markers in pdf format. 3D objects are so-called spatial figures. The criteria for creating 3D objects are based on the "Initial Guides for Improving Spatial Ability with AR Tasks (GISAR)" (Ozcakir & Cakiroglu 2021).

Procedures for using mAR

Students will use the mAR in lessons that incorporate technological and conceptual mathematical activities. **Figure 1** shows an instance of the mAR. Teacher needs to create a GISAR-compliant 3D object using an object construction application before the lesson. 3D objects that satisfy the GISAR problem-setting criteria are composite solids that are a combination of rectangles and triangles and can be created using Tinkercad. Tinkercad combines unique objects to create a wide variety of complex objects. The length of a side of a unique object and the size of the object itself can be scaled up or down. When the teacher inputs the data of the created 3D object into the mAR, an AR marker is automatically issued in pdf format. The image on the right shows the screen of the mobile device used by the students to read the AR marker paper with the mAR. The green object is the AR object that appears on the AR marker when the marker is read. Students can read the specified URL to activate the mAR during the lessons. The system does not have access control. When students use the mAR, the camera is displayed on the web browser. Furthermore, the mAR has a marker-tracking function. Students read the AR marker with the displayed camera, and the AR object is displayed. The object can be rotated freely in any direction.

Figure 1

The mAR Use Instance



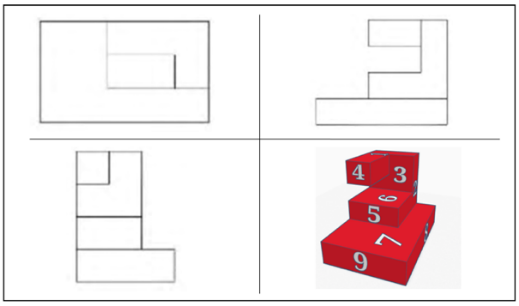
Learning Materials and Spatial Visualization Ability Test

Learning Materials

The GISAR consists of four types of problems, which are the rules when using AR applications. The problems are to identify the faces and vertices of a figure, determine the number of identical elements from a single spatial figure, determine the appropriate plan, front, and side views, and draw a projection. **Figure 2** shows the worksheet for the GISAR-based problems. Students will refer to this question paper and read the AR marker booklets handed out separately to solve the questions.

Figure 2

The problem of Identifying Faces and Vertices



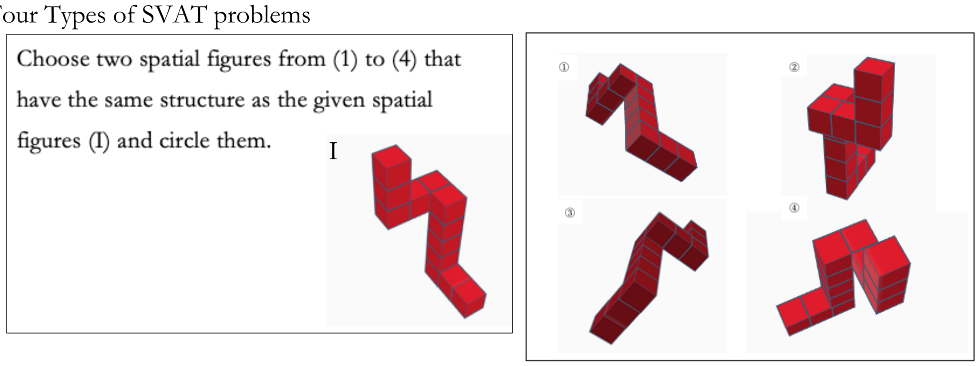
Spatial Visualization Ability Test

We created a test to measure spatial visualization ability based on Mental Rotations Tests (MRT). We named that test Spatial Visualization Ability Test (SVAT). The MRT is used to measure spatial visualization; it consists of 20 items in total, with four types of questions and five figures per question (Vandenberg & Kuse 1978). Each item consists of a reference figure, two correct and two incorrect choices, or distractors. The correct answer choices always have the same structure as the criterion but are presented in a rotated state. The "distractors" are rotated versions of the reference shapes of the other figures, which may be mixed in the question.

The difference between SVAT and MRT is the criterion figure. Since the experimental subjects ranged from first-year junior high school students to first-year high school students, the difficulty level varied greatly from question to question. Instead of providing similar basic figures, different reference figures were provided for each. SVAT suppresses rapid changes in difficulty due to "distraction"**.** Students have ten minutes to solve the SVAT, and the participants must solve the questions independently during the test. **Figure 3** shows one problem with the SVAT.

Figure 3

One of the Four Types of SVAT problems



Conclusion

We discussed mobile-augmented reality (mAR) and learning materials and tests, which are elements of the lessons. We will introduce the mAR in two lessons along with technical and conceptual mathematical activities. Specifically, the technical mathematical activities need to be woven into teaching acts such as geometric constructions and measurements. We believe that conceptual mathematical activities must interweave inductive reasoning, generalization, and proof. When using mAR, teachers need to prepare objects with similar patterns and create multiple inductive proof problems along the way.

Furthermore, the experimental subjects ranged from first-year middle school to first-year high school. Before teaching the technical and conceptual mathematical activities, we should the students' prerequisite knowledge about spatial figures by a performance test. Then, after teaching these activities, I will measure spatial visualization ability using the MRT. In the future, it needs to elaborate on the technical and conceptual mathematical activities and devise two different lesson instruction plans. In addition, it is important to focus on how to use mAR.

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